tangential stress, the equilibrium of the forces, shown by fig. 6a obeys the following relation

$$2\,p_2 k r_1 + 2\int_1^k \sigma_t \,\mathrm{d}\,(l r_1) = 2\,p_1 r_1$$

or

$$\int_1^k \sigma_t \,\mathrm{d}l = -\int_1^k \mathrm{d}\,(pl) = \int_1^k \mathrm{d}\,(\sigma_r l). \tag{1}$$

In the last integral, the pressure p, considered as a positive quantity, has been replaced by $-\sigma_r$ according to the sign convention. σ_r is the radial stress.

Eq. (1) being true whichever value k may have, one can write

$$\sigma_t \,\mathrm{d}l = \mathrm{d}\,(\sigma_r l) \quad \text{or} \quad \sigma_t - \sigma_r = l\,\frac{\mathrm{d}\sigma_r}{\mathrm{d}l}. \tag{2}$$

Eq. (2) is true for the elastic as for the plastic state as far as the deformed wall remains perfectly straight, circular and concentric. If however the deformation of the wall becomes important one must put down $r + u$ into eq. (2) instead of r, because the equilibrium of the forces refers to the deformed wall and not to the shape, the wall has, when it is at rest.

Only considering now the very small elastic deformations, one can write the following relations expressing the tangential strain ε_t and the radial strain ε_r

$$\varepsilon_t = \frac{2\pi\,(r+u) - 2\pi r}{2\pi r} = \frac{u}{r},$$

$$\varepsilon_r = \frac{[(r + \mathrm{d}r + u + \mathrm{d}u) - (r+u)] - \mathrm{d}r}{\mathrm{d}r} = \frac{\mathrm{d}u}{\mathrm{d}r}.$$

ε_t and ε_r are not independent, because after deriving ε_t and eliminating $\mathrm{d}u/\mathrm{d}r$, one finds eq. (3), which is a consequence of the cylindrical symmetry and is called the "comptability equation"

$$\varepsilon_r - \quad = r\,\frac{\mathrm{d}\varepsilon_t}{\mathrm{d}r} = l\,\frac{\mathrm{d}\varepsilon_t}{\mathrm{d}l}. \tag{3}$$

As a rule, it is sufficient to express eq. (3) in terms of the stresses σ_t and σ_r and of the axial stress σ_z and the problem may be now considered as solved, because we dispose of eqs. (2) and (3) and also of an equation, which is not yet written but is the expression, which the forces axial eqiulibrium obeys. For simplifying this problem, one must assume as LAME [1852] did, that the transverse sections of the wall remain plane after deformation.

This assumption has been experimentally confirmed provided these sections are sufficiently remote from the ends of the cylinder considered. A wall, which is submitted to pressures does neither bend nor show any other distorsion than an axial and symetric deformation. We shall consequently put down $\varepsilon_z =$ constant or $d\varepsilon_z/dl = 0$.

One can write now following relations, based on Hooke's law and in which the strains are linear functions of the stresses

$$E\,\varepsilon_t = \sigma_t - \nu\,(\sigma_r + \sigma_z), \tag{4a}$$

$$E\,\varepsilon_r = \sigma_r - \nu\,(\sigma_t + \sigma_z), \tag{4b}$$

$$E\,\varepsilon_z = \sigma_z - \nu\,(\sigma_t + \sigma_r), \tag{4c}$$

E and ν being the Young's modulus and Poisson's ratio.

By making use of eqs. (4a-c) one forms the expressions $\varepsilon_r - \varepsilon_t$ and $\varepsilon_t + \nu\,\varepsilon_z$. which are introduced into eq. (3), the right hand side of which can also be written $d\,(\varepsilon_t + \nu\,\varepsilon_z)/dl$ because ε_z is constant. By doing so, eq. (3) is expressed in terms of the stresses

$$\sigma_t - \sigma_r = \nu l\,\frac{d\sigma_r}{dl} - (1 - \nu)\,l\,\frac{d\sigma_t}{dl}. \tag{5}$$

Eqs. (5) and (2) give each an expression of $\sigma_t - \sigma_r$. One can write by comparing these expressions together : $d\,(\sigma_t + \sigma_r)/dl = 0$, and consequently $\sigma_t + \sigma_r = 2A$. By replacing σ_t by $2A - \sigma_r$ in eq. (2) one finds the value of σ_r after integration

$$\sigma_r = A - Bl^{-2} \tag{6}$$

and also the value of σ_t

$$\sigma_t = A + Bl^{-2}, \tag{7}$$

A and B being two constants still undetermined. On the other side, eq. (4c) shows now, that σ_z is constant

$$\sigma_z = C. \tag{8}$$

This is an interesting result indeed, because the stresses themselves cannot be measured by making adequate experiments to that purpose.

The value of the constants A, B and C, depend upon the case envisaged. As it has been decided, to submit the wall to an internal pressure and an external one, one must put down into eq. (6) $\sigma_r = -\,p_1$ when $l = 1$ and $\sigma_r = -\,p_2$ when $l = k$, so that two particular relations can be made available and made use of, for easily determining the values of constants A and B,